Int. J. Multiphase Flow Vol. 16, No. 4, pp. 567–578, 1990
Printed in Great Britain.

0301-9322/90 $3.00 + 0.00

VISCOUS RESUSPENSION OF A SEDIMENT WITHIN A LAMINAR AND STRATIFIED FLOW

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(*Received* 26 *August* 1988; *in revised form* 23 *October* 1989)

Abstract—Resuspension, under the action of shear, of an initially settled bed of particles has been observed not only in turbulent flows but also under laminar conditions. By applying a model developed previously, we investigate theoretically such a resuspension in a fully-developed Hagen–Poiseuille channel flow and in a corresponding gravity-driven film flow along an inclined plate. It is found that, in the former case, the region occupied by the suspension cannot extend beyond the plane of zero shear stress, while for the film flow, a critical value for the feed concentration is predicted beyond which steady-state operation is no longer possible. Experimental observations in a channel flow are found to be in good agreement with the theory.

Key Words: resuspension, laminar stratified flow

1. INTRODUCTION

As mentioned by Leighton & Acrivos (1986), the conditions under which a settled layer of negatively buoyant particles will resuspend are not well-understood and are usually associated with turbulence at high Reynolds numbers (Thomas 1961). Gadala-Maria (1979) appears to have been the first to notice, however, that such a resuspension can also occur at low Reynolds numbers, for which inertial effects are insignificant and the flow is laminar. Leighton & Acrivos (1986) designated this phenomenon "viscous resuspension" and demonstrated that the equilibrium height that is achieved within a Couette flow can be predicted by a balance of the downward flux of particles due to settling and a Fickian diffusive flux due to the existence of a concentration gradient. These authors neglected Brownian motion and attributed the diffusive flux to the existence of a shear-induced random motion of the particles characterized by a diffusion coefficient proportional to the applied shear rate and to the square of the particle radius (Leighton & Acrivos 1987a). For a plane Couette flow, where the shear stress is constant, Leighton & Acrivos (1986) showed that the change in the interfacial height of the resuspended sediment is proportional to the applied shear stress, and were able to verify their prediction experimentally over a wide range of parameters. Although, their results apply only as long as a sediment layer remains at the bottom of the channel where the particle concentration equals its initial maximum value, this restriction can easily be relaxed. In such a case, however, the height of the suspension layer is no longer linear in the shear rate.

In the present paper we investigate theoretically viscous resuspension for two other shear flows, specifically a two-dimensional (2-D) Hagen–Poiseuille channel flow and a gravity-driven film flow along an inclined plate. In both cases the shear stress is no longer constant and for the Hagen–Poiseuille flow there exists a plane within the channel where it is zero. This fact places an upper bound on the height of the resuspended layer. In addition, for the gravity-driven film flow, a critical value is predicted for the volume concentration of solids in the well-mixed feed suspension, above which the particles settle out and form a sediment layer along the plate that continues to grow as a function of time. Finally, we investigated this phenomenon experimentally in a channel flow and found good agreement between the experimental results and the theoretical predictions.

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2. BASICS

Following the analysis by Leighton & Acrivos (1986), we consider a suspension of negatively buoyant spheres of uniform size in a 2-D duct or in a film flow, as depicted in figures 1a and 1b.

The symbol h_0 denotes the height of the settled bed with maximum particle volume concentration ϕ_0, i.e. the height of the sediment which would be reached if the flow were to be suddenly turned off, h_t is the position of the top of the resuspended layer in the presence of a laminar shear flow with velocity $U(z)$ and h_c denotes the height of the remaining sediment layer at the bottom. The viscosity is denoted by μ and the density by ρ. The subscript m refers to the particle–fluid mixture within the resuspended layer where both the viscosity μ_m and the density ρ_m are functions of the particle volume concentration $\phi(z)$. Moreover, where necessary within the text, the subscripts 1 and 2 will distinguish between the clear fluid and particle properties, respectively. Also, the symbol g refers to the gravitational constant and α is the angle of inclination. Finally, the total space of the 2-D duct is given by $2B$ (figure 1a), while δ refers to the thickness of the downward flowing film according to figure 1b.

The downward flux of particles with radius a in the direction of gravity is given by

$$N_s = \tfrac{2}{9}\phi \frac{a^2 g(\rho_2 - \rho_1)}{\mu_1} f(\phi), \tag{1}$$

i.e. Stokes' law multiplied by the hindrance function $f(\phi)$, which takes into account the presence of other particles within the suspension. This function, which is strongly dependent on the concentration ϕ, has not been measured to date in the presence of shear. Thus, for simplicity we shall take it as being equal to the corresponding function as obtained in vertical settling experiments, which, for our purpose, can be represented approximately by

$$f = \frac{1-\phi}{\mu_r}, \tag{2}$$

where μ_r is the dimensionless effective viscosity of the suspension given by Leighton (1985) as

$$\mu_r = \frac{\mu_m}{\mu_1} = \left[1 + \frac{1.5\phi}{1 - \dfrac{\phi}{\phi_0}}\right]^2. \tag{3}$$

Equation [2] can also be obtained by assuming that the spheres settle with their Stokes velocity in a fluid having the density and viscosity of the suspension. Of course the precise form chosen

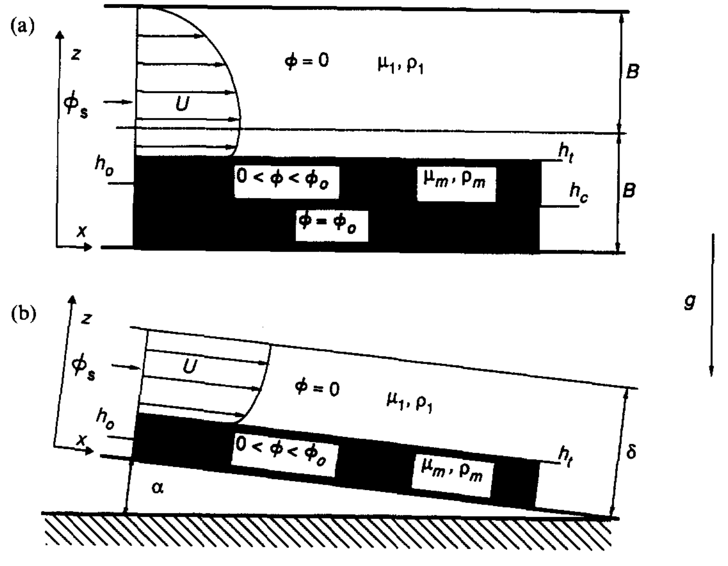

Figure 1. (a) Schematic of a 2-D Hagen–Poiseuille channel flow indicating the notation. (b) Plane film flow.

for $f(\phi)$ has little bearing on the principal features of the solution as long as $f(\phi)$ decreases monotonically with increasing particle concentration.

In the presence of a concentration gradient, the Fickian diffusive flux

$$N_d = -D\frac{d\phi}{dz}, \qquad [4]$$

(with D being the shear-induced diffusion coefficient) balances the flux due to sedimentation, and hence we obtain

$$\tfrac{2}{9}\phi f\frac{a^2 g(\rho_2-\rho_1)}{\mu_1} + \dot{\gamma}(z)a^2\hat{D}\frac{d\phi}{dz} = 0, \qquad [5]$$

where $\hat{D}$ denotes the dimensionless diffusion coefficient

$$\hat{D} = \frac{D}{\dot{\gamma}a^2},$$

with $\dot{\gamma}$ being the absolute value of the local shear rate

$$\dot{\gamma} = \frac{\tau(z)}{\mu_m(z)} \qquad [6]$$

and $\tau(z)$ the shear stress. An approximate estimate for the diffusion coefficient is

$$\hat{D} \approx \tfrac{1}{3}\phi^2(1 + \tfrac{1}{2}e^{8.8\phi}), \qquad [7]$$

which was found by Leighton (1985) to represent his data. We note parenthetically that the proportionality between D and $\dot{\gamma}$, which applies only for 2-D unidirectional flows, has not been extended to date to more complicated situations. Also, for the sake of simplicity, we have neglected the particle flux due to gradients in the shear stress (Leighton & Acrivos 1987b) which would have added a second term to the r.h.s. of [4].

Equations [2], [3] and [5]–[7], plus the equations of motion, consist of a set of equations which together with the appropriate boundary conditions describe the flow field. In the following two sections we shall consider separately the resuspension problem for a Hagen–Poiseuille channel flow and for a plane film flow along an inclined plate.

3. HAGEN–POISEUILLE CHANNEL FLOW

Consider the fully-developed laminar stratified flow of an initially, well-mixed suspension, having a particle volume fraction ϕ_s, flowing along a 2-D channel, as shown in figure 1a.

We begin our analysis by introducing dimensionless variables using $2B$ and $Q/2B$ as the characteristic length and velocity, respectively, where Q is the volume flux of clear liquid per unit depth. The pressure is scaled with $\mu_1 Q/4B^2$. Then, on integrating the reduced equations of motion for a fully-developed flow subject to the no-slip boundary conditions at the wall and the requirement that the velocity and shear stress be continuous at the interface, we obtain for the velocity within the clear fluid,

$$U_1 = K[\tfrac{1}{2}(1-z^2) - C(1-z)]; \qquad [8]$$

and for the velocity within the resuspended layer,

$$U_m = K\left[\int_{\lambda}^{z}\frac{1}{\mu_r(\tilde{z})}(C-\tilde{z})d\tilde{z}\right]; \qquad [9]$$

where

$$C = \frac{\dfrac{1}{2} - \dfrac{h_t^2}{2} + \displaystyle\int_{\lambda}^{h_t}\frac{z\,dz}{\mu_r(z)}}{1 - h_t + \displaystyle\int_{\lambda}^{h_t}\frac{dz}{\mu_r(z)}}. \qquad [10]$$

In the above, K is the dimensionless pressure-drop coefficient, defined by

$$\Delta p = p_{\text{in}} - p_{\text{out}} = KL,$$

where L is the dimensionless length of the duct. Also, $\lambda = h_c$ if $h_c \geq 0$, while $\lambda = 0$ if the whole sediment has resuspended.

The remaining two equations which are needed to determine K and h_t are the global mass balance of the clear fluid for a given flow rate,

$$\int_{h_t}^{1} U_1(z)\mathrm{d}z + \int_{\lambda}^{h_t} [1 - \phi(z)]U_m(z)\mathrm{d}z = 1, \qquad [11]$$

and the solids material balance,

$$\frac{\phi_s}{1 - \phi_s} = \int_{\lambda}^{h_t} \phi U_m \, \mathrm{d}z, \qquad [12]$$

where, as mentioned above, ϕ_s refers to the particle volume fraction in the well-mixed suspension entering the channel.

Finally, we have for the dimensionless shear rate within the resuspended layer

$$\dot{\gamma} = \frac{\mathrm{d}U_m}{\mathrm{d}z} = \frac{K}{\mu_r}(C - z), \qquad [13]$$

which when substituted into [5] and together with [2] leads to

$$\phi(1 - \phi) = \kappa K(z - C)\hat{D}\frac{\mathrm{d}\phi}{\mathrm{d}z}. \qquad [14]$$

The symbol

$$\kappa = \frac{3}{8}\frac{a}{B}\psi \qquad [15a]$$

is a modified Shields number,

$$\psi = \frac{3}{2}\frac{\mu_1 Q}{aB^2 g(\rho_2 - \rho_1)} \qquad [15b]$$

(Leighton & Acrivos 1986), which gives a measure of the ratio of viscous forces to those of gravity in the bulk flow. Note that $h_t \leq C$, owing to the fact that the shear-induced diffusion coefficient vanishes along the plane of zero shear stress $z = C$.

In the neighborhood of the interface (or, for sufficiently large values of κK, throughout the whole resuspended layer if the height of the settled bed is small), the limiting case $\phi \ll 1$ applies. Thus, the diffusion coefficient becomes simply

$$\hat{D} = \tfrac{1}{2}\phi^2, \qquad [16]$$

which when substituted in [14] yields, on integration,

$$\phi^2 = \frac{4}{\kappa K}\ln\frac{C - z}{C - h_t} \qquad \text{as } z \to h_t. \qquad [17]$$

In general, when $\kappa K \to \infty$, the concentration reaches a constant value ϕ^* within the resuspended layer and $h_t = C$. Consequently, in view of [10] and the fact that $h_c = 0$, implying $\lambda = 0$, and after some algebraic manipulations involving [11] and [12], we obtain that

$$\frac{\phi^*}{\phi_s} = \frac{1}{h_t} = 1 + \frac{1}{\sqrt{\mu_r(\phi^*)}} \qquad [18]$$

and

$$K = \frac{3}{(1 - \phi_s)\left(1 - \dfrac{\phi_s}{\phi^*}\right)^2}. \qquad [19]$$

We remark parenthetically that, as $\kappa K = \rightarrow\infty$, the slope of the base velocity profile vanishes on the interface since $C = h_t$. This has important implications regarding the linear stability analysis of such a flow because the interfacial mode is neutrally stable under these conditions (Yiantsios & Higgins 1988).

We also indicate for further use, that h_0, the height of the sediment which would be reached if the flow were to be suddently turned off, can be determined once the concentration profile has been found by making use of the global conservation of particles equation,

$$\int_{\lambda}^{h_t} \phi \, dz = \phi_0(h_0 - h_c). \qquad [20]$$

As can be verified by direct substitution, the system of equations [8]–[12] and [14] can be simplified via the transformations

$$K = \frac{M}{\kappa}, \quad h_t = 1 - N\kappa^{1/3}, \quad h_c = 1 - P\kappa^{1/3}, \quad h_0 = 1 - R\kappa^{1/3}, \quad C = 1 - S\kappa^{1/3}, \quad z = T\kappa^{1/3}, \qquad [21]$$

where the various coefficients M, N etc. are functions only of ϕ_s. This transformation applies, of course, only if $\kappa^{1/3} \leqslant 1/P$. Thus, as was the case in Couette flow (Leighton & Acrivos 1986), there exists a simple functional relation between $\Delta h = h_t - h_c$ and κ as long as a sediment layer is present at the bottom of the channel, except that here, Δh is linear in $\kappa^{1/3}$ rather than in κ.

Off hand, it might seem that the system of equations [8]–[12] and [14] is so simple that any standard numerical technique could have been employed for the purpose of obtaining a solution. Unfortunately, [14] is stiff in that, as can be seen in figure 2, ϕ changes very rapidly with position near the suspension–clear fluid interface, but is otherwise nearly constant over the rest of the suspension layer. Consequently, a way had to be devised in order to circumvent this difficulty.

First of all, since [14] is separable, it can be integrated to yield

$$\frac{3}{\kappa K}\ln(1 + y) = F(\phi) \equiv \int_0^{\phi} \frac{t}{1-t}\,(1 + \tfrac{1}{2}e^{8.8t})dt, \qquad [22]$$

$$y \equiv \frac{h_t - z}{C - h_t},$$

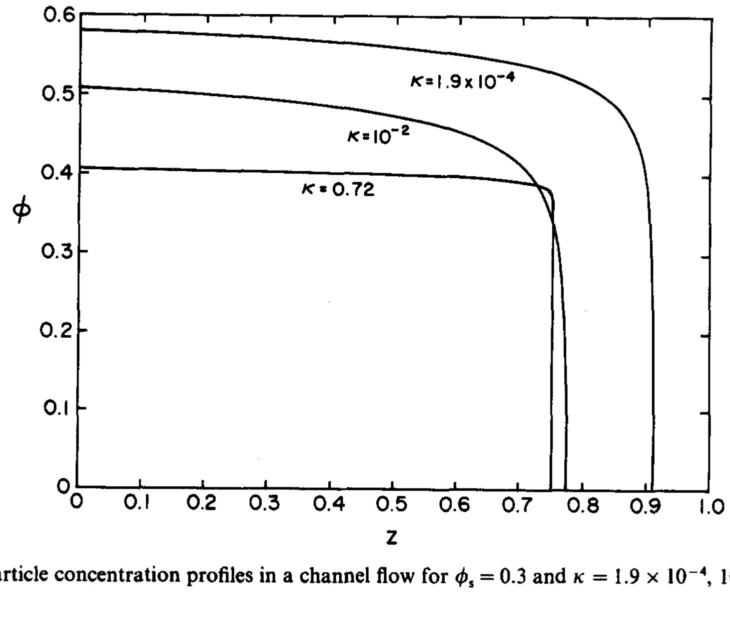

Figure 2. Particle concentration profiles in a channel flow for $\phi_s = 0.3$ and $\kappa = 1.9 \times 10^{-4}$, 10^{-2} and 0.72.

where use has been made of the expression for $\hat{D}$ given in [7]. Then, if $h_c > 0$, we have from [22] that

$$\frac{C - h_c}{C - h_t} = \exp\left[\frac{\kappa K F(\phi_0)}{3}\right], \qquad [23]$$

where, as before, ϕ_0 equals the maximum particle concentration. On the other hand, if $h_c = 0$,

$$\frac{C}{C - h_t} = \exp\left[\frac{\kappa K F(\phi_c)}{3}\right], \qquad [23a]$$

where ϕ_c is the unknown particle concentration at $z = 0$. Thus, [23] or [23a] gives one relation among the three unknowns, h_t, h_c, and C (or h_t, C and ϕ_c), in terms of κK.

Next, [10] is recast into

$$\frac{C - h_t}{1 - h_t} \equiv \Theta = \frac{\dfrac{1}{2} - \Theta^2 \displaystyle\int_0^{\hat{\phi}} \frac{y}{\mu_r} \frac{\mathrm{d}y}{\mathrm{d}\phi}\,\mathrm{d}\phi}{1 + \Theta \displaystyle\int_0^{\hat{\phi}} \frac{1}{\mu_r} \frac{\mathrm{d}y}{\mathrm{d}\phi}\,\mathrm{d}\phi}, \qquad [24]$$

where $\hat{\phi}$ equals either ϕ_0 (if $h_c > 0$) or ϕ_c, while [11], [12] and [20] become, respectively:

$$\frac{1}{3} - \frac{C}{2}(1 - h_t)^2 - \tfrac{1}{6} h_t(3 - h_h^2) + C - h_t)^3 \int_0^{\hat{\phi}} (1 - \phi) A(\phi, \hat{\phi})\,\mathrm{d}\phi = \frac{1}{K}, \qquad [25]$$

$$\frac{\phi_s}{1 - \phi_s} = K(C - h_t)^3 \int_0^{\hat{\phi}} \phi A(\phi, \hat{\phi})\,\mathrm{d}\phi \qquad [26]$$

and

$$(C - h_t) \int_0^{\hat{\phi}} \phi \frac{\mathrm{d}y}{\mathrm{d}\phi}\,\mathrm{d}\phi = \phi_0(h_0 - h_c), \qquad [27]$$

with

$$A(\phi, \hat{\phi}) \equiv \frac{\mathrm{d}y}{\mathrm{d}\phi} \int_{\phi}^{\hat{\phi}} \frac{1 + y}{\mu_r(\phi')} \frac{\mathrm{d}y}{\mathrm{d}\phi'}\,\mathrm{d}\phi'.$$

Thus, given κ and ϕ_s, it is a simple matter to determine, from [23]–[27], the unknowns, C, K, h_t, h_0 and h_c (or ϕ_c).

In all our calculations ϕ_0 was set equal to 0.58, the value inferred from Leighton's (1985) experiment.

Shown in figure 2 are some typical concentration profiles which clearly illustrate that, as noted earlier, ϕ changes very rapidly near the suspension–clear fluid interface, and hardly at all below it. Of particular interest from the practical point of view, however, is figure 3 which shows the pressure-drop coefficient K, relative to its value for a pure fluid, plotted vs κ, with ϕ_s as a parameter. It is seen that the two asymptotic expressions for K noted above when $\kappa^{1/3} \leqslant 1/P$ or $\kappa \to \infty$, respectively (cf. [21] and [18, 19]) suffice to describe the functional dependence of K over practically the whole range of κ.

A quantity that can readily be measured experimentally is h_0, the height of the settled sediment following a shut down of the flow, which is shown in figure 4 plotted as a function of κ and ϕ_s.

4. PLANE FILM FLOW

We next turn to the problem of determining the fully-developed flow of an initially well-mixed suspension down an inclined wall, as shown in figure 1b. By referring all lengths to a film thickness

$$\delta^* = \left(\frac{3Q\nu}{g \sin\alpha}\right)^{1/3},$$

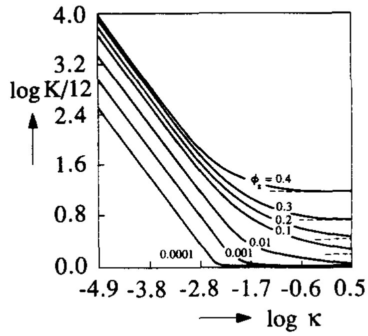

Figure 3. Pressure drop coefficient $K/12$ vs κ for different values of ϕ_s.

which is set up by a volume flux Q of clear liquid with kinematic viscosity ν, and by non-dimensionalizing the shear stress with respect to $g\rho_1\delta^*\sin\alpha$ and the velocities by $g\nu^{-1}(\delta^*)^2\sin\alpha$, we immediately obtain, by applying a dimensionless force balance, that

$$\tau_m = (\delta - z) + \epsilon \int_z^{h_t} \phi \, dz, \qquad [28]$$

where

$$\epsilon = \frac{\rho_2 - \rho_1}{\rho_1}$$

and δ is the dimensionless thickness of the whole film (figure 1b). Then, on substituting [2] and [28] into [5], we obtain for the dimensionless flux balance:

$$\frac{2}{9}\phi(1-\phi) + \left[\int_z^{h_t}(1+\phi\epsilon)dz + (\delta - h_t)\right]\frac{\hat{D}\tan\alpha}{\epsilon}\frac{d\phi}{dz} = 0. \qquad [29]$$

In addition, the dimensionless velocity within the resuspended layer is given by

$$U_m = \int_0^z \frac{\tau_m}{\mu_r} d\tilde{z}, \qquad 0 \leqslant z \leqslant h_t, \qquad [30]$$

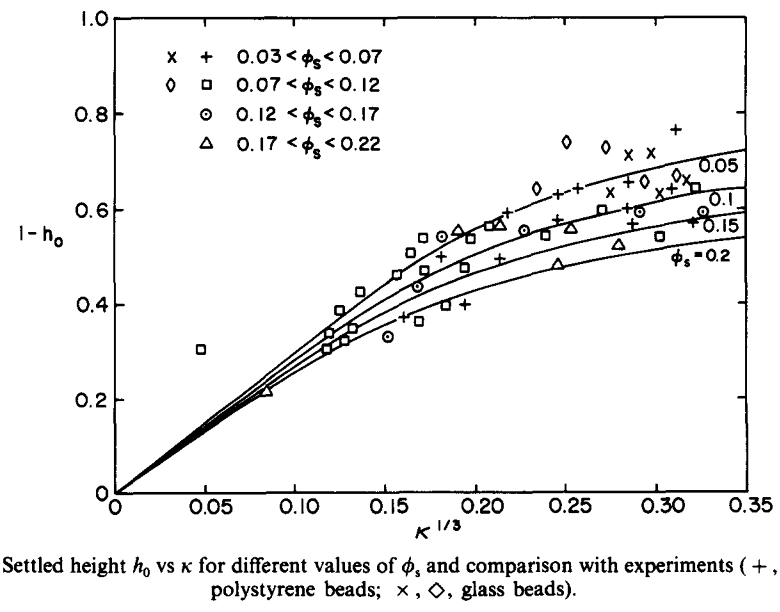

Figure 4. Settled height h_0 vs κ for different values of ϕ_s and comparison with experiments (+, □, ○, △, polystyrene beads; ×, ◇, glass beads).

and the velocity within the clear fluid above the resuspended layer becomes

$$U_1 = \tfrac{1}{2}(h_t^2 - z^2) - \delta h_t + \delta z + \int_0^{h_t} \frac{\tau_m}{\mu_r}\,dz. \qquad [31]$$

Finally, the thickness of the film δ follows from a global mass balance of the clear fluid,

$$\int_0^{h_t} (1-\phi)U_m\,dz + \int_{h_t}^{\delta} U_1\,dz = \tfrac{1}{3}, \qquad [32]$$

while a mass balance on the particles gives

$$\frac{\phi_s}{1-\phi_s} = 3\int_0^{h_t} \phi U_m\,dz + \frac{2}{3}\left(\frac{a}{\delta*}\right)^2 \epsilon \int_0^{h_t} \phi f(\phi)dz. \qquad [33]$$

Clearly, the last term in [33] is negligible for all systems, which can be represented as effective continua and henceforth will be omitted.

This system of equations describing resuspension in a film flow is very similar to that discussed earlier in connection with flow in a channel, and hence was solved in like fashion. All calculations to be reported below were performed for $\epsilon = 1$ and $\phi_0 = 0.58$.

Figure 5 depicts the quantity δ, i.e. the thickness of the film, as a function of the particle concentration ϕ_s in a well-mixed suspension upstream, for various values of the angle of inclination α.

It should be noted that, in contrast to the Hagen–Poiseuille case where the channel width is fixed and a pressure gradient to be determined as part of the solution is set up, a fully-developed laminar film flow can exist here only as long as ϕ_s is below some critical value ϕ_s^* which depends only on α and ϵ. To see this, consider first the case $\phi_s \to 0$ which implies that ϕ is everywhere small. Consequently, the particle concentration along the plate $z = 0$ is lower than ϕ_0 and a sediment layer will not exist. When the feed concentration is increased, however, the thickness of the film will increase because the increase in the effective viscosity will lower the strength of the shear flow which maintains the shear-induced particle diffusion. Although, in view of [7], this will be compensated at first by a corresponding increase in $\hat{D}$, a point will eventually be reached when a sediment layer will form on the surface of the plate. This layer will then continue to grow as a function of time because the applied shear flow will be of insufficient strength to prevent some of the particles that are being supplied in the feed from continuously settling out to form a stagnant sediment. The existence of such a critical feed concentration ϕ_s is clearly seen in figure 5, where the dashed curve represents the loci of the point in the (δ, α) plane at which the particle concentration along the plate $z = 0$ first equals $\phi_0 = 0.58$, the particle concentration of a settled bed. Although as it is turns out,

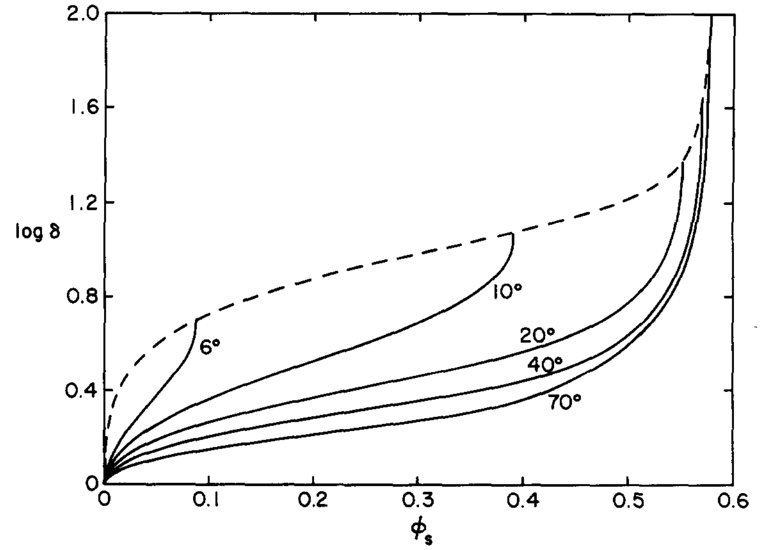

Figure 5. Thickness δ vs ϕ_s for different values of α.

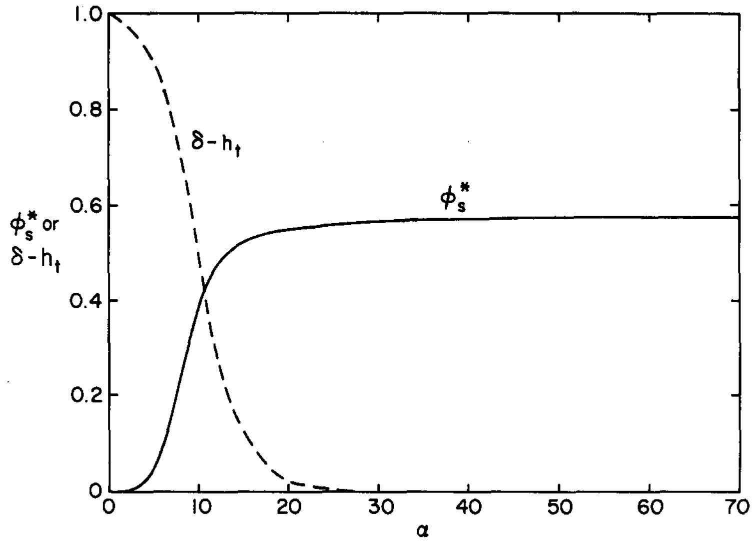

Figure 6. The critical values of ϕ_s and $(\delta - h_t)$ vs α at the limit for steady film flow.

solutions to the system of equations given above still exist, for any given α, beyond the dashed curve due to the particular form chosen for the effective viscosity μ_r, [3], these solutions lack physical significance because they yield values for ϕ at $z = 0$ which are larger than $\phi_0 = 0.58$.

Figure 6 shows the dependence on α of ϕ_s^*, the critical value of ϕ_s, beyond which a steady solution to our model equations describing film flow does not exist, as well as the corresponding values of $\delta - h_t$, while the corresponding concentration profiles are depicted in figure 7.

5. EXPERIMENTS

A sketch of the experimental apparatus is shown in figure 8. The test section consisted of a rectangular duct with cross section 80 × 10 mm and length 1 m. The conduit had inlet ports for the sediment and the clear fluid, and an outlet port for the suspension. The concentration ϕ_s of the negatively buoyant particles (glass beads with radius $a = 65\ \mu m$ and density $\rho_2 = 2480\ kg/m^3$; or polystyrene beads with radius $a = 444\ \mu m$ and density $\rho_2 = 1054\ kg/m^3$) was measured by means of a laser beam, a transparent cell and a photo diode, a technique which has been described

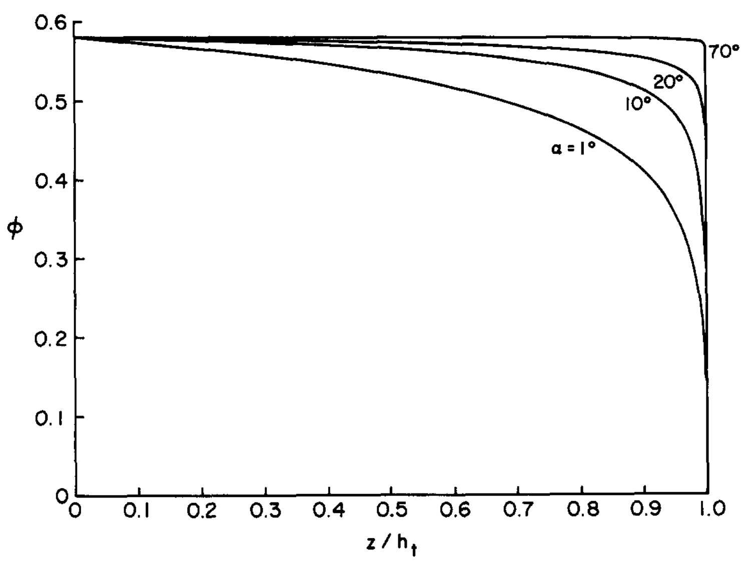

Figure 7. Particle concentration profiles in a film flow for various angles of inclination α when ϕ_s equals its critical value ϕ_s^*.

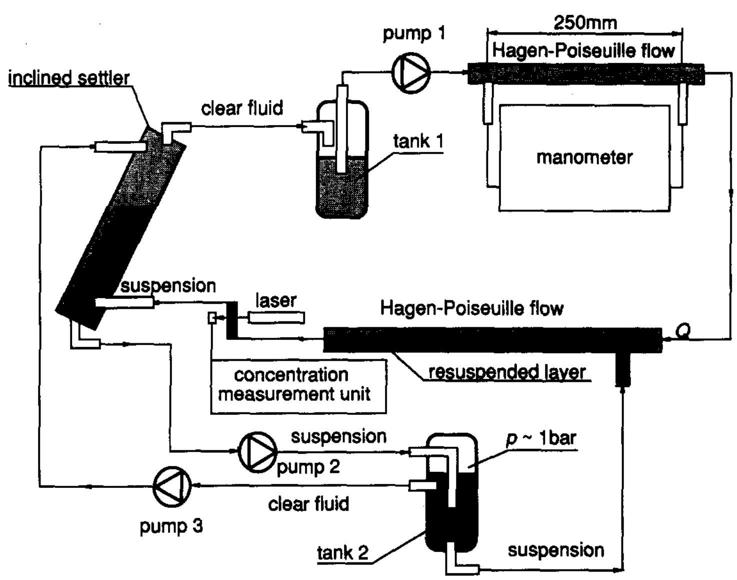

Figure 8. Experimental setup.

in more detail by Borhan (1989). Subsequently, the particles were separated from the fluid (water–glycerine mixture, $\rho_l = 113\ \mathrm{kg/m^3}$; or water–ethyl alcohol mixture, $\rho_l = 822\ \mathrm{kg/m^3}$) within an inclined settler. Pump 2 conveyed the highly concentrated suspension into tank 2, from which a sediment with maximum concentration ϕ_0 was forced back into the resuspension conduit by means of an adjustable pressure (< 1 bar). Pump 3 returned the surplus of the clear fluid into the settler from which the purified liquid flowed into tank 1, where air bubbles could escape. Finally, by using pump 1, the clear fluid was made to flow through a manometer and to return into the resuspension conduit.

By suitably setting the flow rate of the clear liquid and the pressure difference between tank 2 and the resuspension conduit, it was found possible to achieve a steady equilibrium resuspension height within a short time. This height h_t, as well as the settled height h_0 after a sudden shut down of all the flows, were measured at different locations by means of a cathetometer.

Unfortunately, the appearance of instabilities within the rectangular duct limited the range of the experiments. Specifically, for glass beads suspended in a water–glycerine mixture, it was difficult to maintain a constant resuspended height within the duct if $h_0 > 0.3$. In that case instabilities were noted and a certain portion of the cross section became completely blocked by sediment, with other areas totally free of particles. These observations provided the motivation for a linear spatial stability analysis which will be described in a subsequent paper (Schaflinger & Acrivos 1990).

The experimental data for h_0 as a function of κ and ϕ_s are compared with the theory in figure 4, and the corresponding comparison of the resuspension data is shown in figure 9. Evidently,

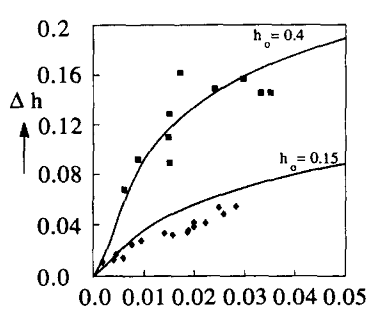

Figure 9. Resuspension data for a 80 × 10 mm rectangular duct. ■, polystyrene beads; measured settled height, $0.35 < h_0 < 0.45$. ◆, glass beads, measured settled height, $0.12 < h_0 < 0.17$. The lines correspond to the respective theoretical calculations.

there is good agreement between the theory and the experiments, especially considering that the theory contained no adjustable parameters.

6. CONCLUSIONS

In most sediment transport problems, the flow is highly turbulent and viscous resuspension plays only a minor role, except in the viscous sublayer adjacent to the boundaries of the flow. Nevertheless, shear-induced particle diffusion and the resulting resuspension may be of importance in many laminar flows, such as in cross-flow microfiltration (Davis & Leighton 1987) and the sediment transport in inclined settlers.

In the present paper, we investigated theoretically the viscous resuspension of negatively buoyant particles in a fully-developed 2-D Hagen–Poiseuille flow and in a corresponding plane, gravity-driven film flow. The theoretical approach was based on a model developed by Leighton & Acrivos (1986) in which the net downward flux of particles due to gravity is balanced by a diffusive flux caused by a shear-induced random motion of the particles. The model is free from adjustable parameters but does not take into account Brownian motion, which is negligible within the range of particle diameters investigated. It was shown that, in a 2-D Hagen–Poiseuille flow, resuspension is restricted due to the existence of a plane with vanishing shear stress. Moreover, for large values of Shields number, a constant concentration within the resuspended layer is reached and the velocity has its maximum at the interface. This fact is important in the linear stability analysis of the flow because the interfacial mode becomes neutrally stable under these conditions (Yiantsios & Higgins 1988). The pressure-drop coefficient for a 2-D Hagen–Poiseuille flow was also calculated as a function of a modified Shields number and of the particle concentration in a well-mixed suspension far upstream at the entrance region of the duct. Also the settled height in such a flow was plotted as a function of the same parameters.

For the film flow, the total thickness of the film was calculated as a function of ϕ_s, the fraction of solids in a well-mixed suspension, with the angle of inclination and the relative density difference between the solids and the fluid as parameters. It was found that there exists a critical value of ϕ_s above which the particles will settle out and form a sediment layer at the bottom that will continue to grow as a function of time.

Experiments performed in a 2-D Hagen–Poiseuille channel flow gave good agreement between the measurements and the theoretical predictions which contained no adjustable parameters.

Acknowledgements—U. Schaflinger thanks the Max Kade Foundation for a scholarship which enabled him to spend a year at Stanford University. This research was also partially supported by grants from the U.S. Department of Energy (DE 13328) and the National Science Foundation (CTS-8896170).

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